Math & Music Fusion: Teacher Answer Key & Worked Solutions

Grades 6–8 | Teacher Reference
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This key shows **one clear, mathematically sound solution path** for each exercise. Some questions allow for multiple reasonable answers; notes are included where flexibility is appropriate.

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1. Tempo (BPM): Music's Speedometer

1A. Percentage Change

Problem: 140 BPM slowed by 25%

Work:
$140 \times 0.75 =$ **105 BPM**

1B. Beats to Seconds

Problem: 90 BPM for 40 seconds

Work:
90 beats / 60 seconds = 1.5 beats per second
$1.5 \times 40 =$ **60 beats**

1C. Reverse Thinking

Problem: 72 BPM feels like 144 BPM

Answer: The "heartbeat" is moving faster than you think.
$72 \times 2 =$ **144 BPM**

2. Extreme BPM Logic (DJ Math)

2A. Halved or Doubled?

Problem: 220 BPM heard

Work:
220 ÷ 2 = **110 BPM** (common pop tempo)

Answer: Divide by 2; corrected BPM = **110.**

Discuss the ways in how we count where we think the heartbeat is (BPM) are influenced by the **pattern/rhythms** of the song.

2B. Tripled Trouble

Problem: 180 BPM should feel like 60 BPM

Work:
180 ÷ 3 = **60 BPM**

2C. Choose the Best Fix

Problem: 300 BPM

Work:
300 ÷ 2 = 150 BPM (still very fast)
300 ÷ 3 = **100 BPM** (typical pop tempo)

Answer: ÷3 is the better musical choice (but this is **relative** to the listener, the composer, etc).

3. Beat Subdivision: Dividing Time

3A. Counting Notes

Problem: 100 BPM, eighth notes

Work:
Each beat = 2 eighth notes
$100 \times 2 =$ **200 eighth notes per minute**

3B. Comparison

Problem: Which has more notes?

- 80 BPM with sixteenths: $80 \times 4 =$ **320 notes**

- 160 BPM with eighths: $160 \times 2 =$ **320 notes**

Answer: They are **equal**

3C. Real-World Feel

Expected Answer: Sixteenth notes feel faster and denser because **more notes occur in the same amount of time**, even if BPM stays the same.

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4. Patterns = Groove

4A. Pattern Math

Pattern: X – – X | – X – – | X – – – | – X – –

Correct answer:

- Beats **1, 4, 6, 9, and 14 (There are 5 total sounds.)**

What do you notice about how often the sound repeats? Acceptable observations include:

- The sounds are **not evenly spaced**

- There are **groups of 2–3 silent beats** between sounds

- Some sounds happen **closer together**, others farther apart

- The pattern feels **intentional**, not random

Teacher note: There is no single "right" observation here—the goal is noticing structure.

4B. Predict the Next Measure

Correct answer:

- Beats 17, 20, 22, 25, and 30

(These correspond to beats 1, 4, 6, 9, and 14 in the original pattern.)

Explain how you know.

Correct explanation (student-friendly):

- The pattern repeats every **16 beats**

- Add **16** to each beat that had sound:

    ○ 1 → 17

    ○ 4 → 20

    ○ 6 → 22

    ○ 9 → 25

- ◦ 14 → 30

Key idea:
Beats that are **16 apart** land in the same place in the pattern.

4C. Pattern Design

Answer: Any valid 8-beat pattern with exactly 3 hits (e.g., X – – X – – X –)

One correct example:

```
1 2 3 4 5 6 7 8
X – – X – – X –
```
(Sounds on beats **1, 4, and 7**)

Explanation: What makes it predictable?

Strong answers include:

- The sounds are **evenly spaced**
- A sound happens **every 3 beats**
- Once you hear the first few beats, you can **guess when the next sound will happen**
- The spacing follows a **clear rule**

5. Ratios & Polyrhythms

5A. Ratio Match

Problem: 4 hits vs 6 hits

Work:
4 : 6 → simplified → **2 : 3**

5B. Least Common Multiple

Problem: 3-beat and 4-beat cycles

Work:
LCM of 3 and 4 = **12 beats**

5C. Physical Try-It

Answer: Hands line up every **6 beats** (LCM of 2 and 3)

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6. DJ Mixing = Applied Math

6A. Simple Match

Problem: 120 BPM & 90 BPM

Work:
90 × (4/3) = **120 BPM**

Answer: Speed up Song B

6B. Meet in the Middle

Problem: 140 BPM & 100 BPM

Valid Answers: 120 BPM or 110 BPM
(Multiple solutions acceptable if justified)

6C. Justify Your Choice

Expected Answer: Extreme tempo changes distort pitch, groove, and feel — math can be correct but **musically unpleasant**.

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7. Prediction: Musical Foresight

7A. Pattern Prediction

Problem: Repeats every 6 beats, currently on 18

Work:
18 + 6 = **24**

7B. Counting Ahead

Problem: Changes every 8 beats, last at 32

Work:
32 + 8 = **40**

7C. DJ Awareness

Expected Answer: DJs must anticipate transitions to align beats, drops, and changes smoothly.

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8. Follow the Rules

8A. Given Rules

Answer:

Correct 16-beat pattern:

```
1  2  3  4 | 5  6  7  8 | 9 10 11 12 |13 14 15 16
X  –  –  – | X  –  –  – | X  –  –  – | X  –  –  –
```

8B. Count the Results

Answer:
4 claps

Teacher note: This reinforces that rules directly control outcomes.

8C. Change One Rule

Correct observations include:

- One clap **moves earlier or later**

- The **spacing between claps changes**

- The **total number of claps stays the same** (if only one number changes)

- The pattern **feels different**, even though it's still rule-based

➡️ Why this works:
They *experience* rule-based thinking without inventing it from scratch.

Teacher Tip

There may be **multiple musically valid answers**. Prioritize: - Correct math - Clear reasoning - Logical musical justification

This mirrors real-world DJ and music-production decision-making.